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# Plastic deformation – its role in fatigue crack propagation

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Recognizing the fact that the effective driving force ($\Delta K_{\text{eff}}$) determines the fatigue crack propagation (FCP) rate and that the shear strain, which is considered to develop due to an occurrence of crack closure, primarily contributes to the plastic deformation, an effort is made here to elucidate the role of plastic deformation in FCP by developing a correlation between the $\Delta K_{\text{eff}}$ and the applied driving force ($\Delta K$) with shear strain as variable. The effect of the degree of plastic deformation (i.e. shear strain level) on the FCP rates at higher values of $\Delta K$, where $\Delta K_{\text{eff}}$ approaches $\Delta K$, approaching the Paris regime, appears minimal. On the other hand, the disparity between $\Delta K_{\text{eff}}$ and $\Delta K$, which apparently increases with shear strain level, persists at lower values of $\Delta K$. This suggests a strong influence of the degree of localized deformation on the FCP rates in the near threshold level. Hence, an improvement of FCP rates in the near threshold level should follow an effort that promotes the plastic deformation near the crack tip to a greater degree. This approach could, therefore, form the basis to explain the effect of the grain size, microstructure, environment, $R$-ratio and crack size on the near-threshold FCP rates.

## 1. Introduction

Fatigue crack propagation (FCP), in view of its engineering significance, has received a great deal of attention both theoretically and experimentally. The FCP rate ($\mathrm{d}a/\mathrm{d}N$) data are generally plotted against the stress intensity range ($\Delta K$) in which three FCP rate regions are widely recognized as shown schematically in Fig. 1. The regions of interests are I and II. Region II represents the steady state which correlates well with the Paris FCP rate equation. The region I is the near-threshold level. Here, there exists a limiting value of $\Delta K$ corresponding to the threshold stress intensity range ($\Delta K_{\text{th}}$) below which the crack remains almost dormant. The FCP rates in region I are lower than that implied by the extrapolation of the Paris equation, thereby indicating a variation of the FCP rate sensitivities in regions I and II. This is despite the fact that a prerequisite for the FCP is the plastic deformation [1]. The objective of this work is, therefore, to examine the role of plastic deformation in determining the FCP rate sensitivities.

Since plastic deformation directly results from the shear strain, the above objective is accomplished by evaluating the role of shear strain which is considered to develop near the crack tip due to crack closure. This is the critical assumption in this work, the basis of which may lie with the need for higher applied driving force that is actually required for propagating a crack in the event of the occurrence of crack closure.

The consideration of the contribution of the crack closure phenomenon on the crack growth kinetics could also explain the FCP rate sensitivities noted above. The premature closure of crack during the unloading portion of the cycle can arise in plane stress [2] as well as in plane strain [3] conditions. The crack tip driving force is accordingly reduced from the nominally applied value ($\Delta K = K_{\max} - K_{\min}$, where $K_{\max}$ and $K_{\min}$ are the stress intensity factors corresponding to maximum and minimum loads) to some effective value ($\Delta K_{\text{eff}} = K_{\max} - K_{\text{cl}}$, where $K_{\text{cl}}$ is stress intensity to close the crack). The crack closure phenomenon can, therefore, reduce the FCP rate significantly in view of the increasing dependence of FCP rate on $\Delta K_{\text{eff}}$. It is now well documented [3] that the effect of closure is significant at lower values of $\Delta K$ (in region I approaching the threshold) while, it is minimal at higher values of $\Delta K$ (in region II). It then follows that the $\Delta K_{\text{eff}}$ is lower than the $\Delta K$ at lower values of $\Delta K$ and approaches the $\Delta K$ at higher values of $\Delta K$. This explains the existence of the variations of the FCP rate sensitivities in the regions I and II. Thus, development of a correlation between $\Delta K_{\text{eff}}$ and $\Delta K$ with shear strain as variable may provide a basis by which the FCP rate sensitivities, noted above, may be ascertained. This is modelled as follows.

## 2. Model

During crack propagation, it is well known that a fraction ($X_0/R_p$) of the cyclic plastic zone (CPZ) fractures in $\Delta N$ cycles, where $R_p$ is the CPZ size and $X_0$ denotes the incremental crack size in $\Delta N$ cycles. Of relevance then is the strain $\varepsilon(X_0)$ in the element at $X_0$. For a strain (tensile) distribution, $\varepsilon(X)$, of the form [4]:

$$\varepsilon(X) = A/(B + X)^q, \tag{1}$$

$\varepsilon(X_0)$ may be defined as [5]

$$\varepsilon(X_0) = \frac{\alpha}{1 - R}\left(\frac{1}{R_p}\right)^q \Delta\varepsilon \tag{2}$$

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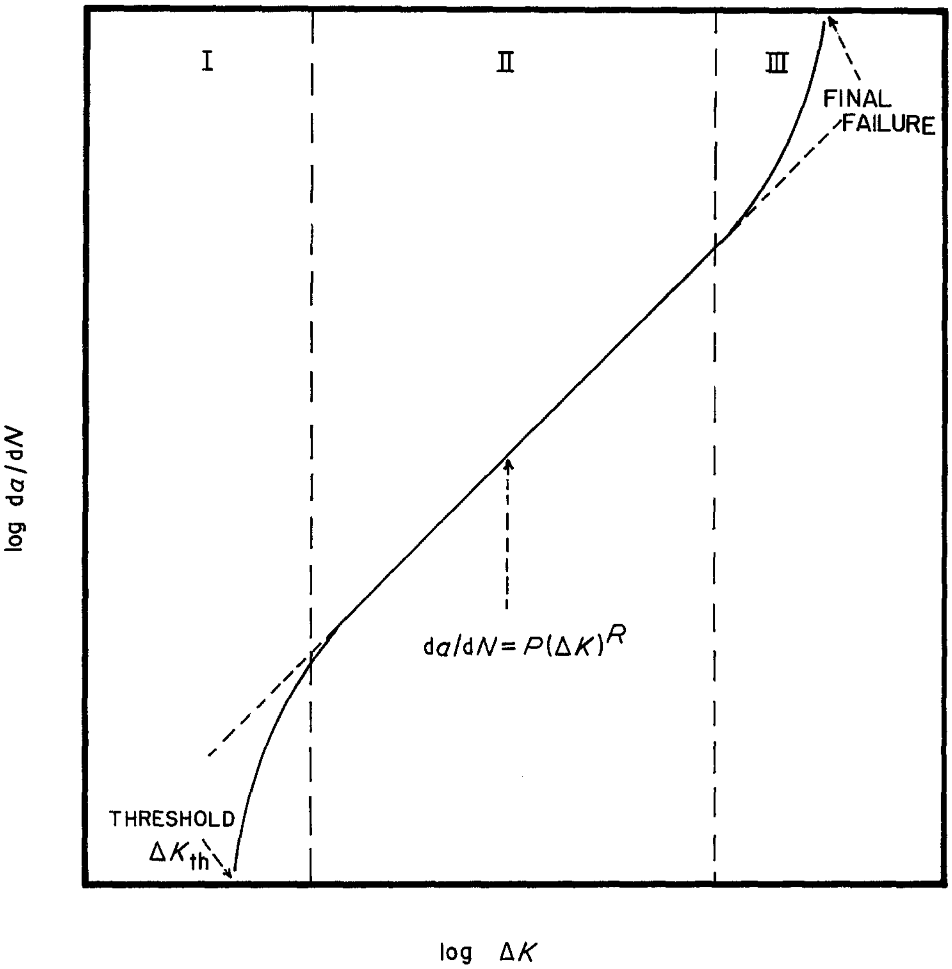


*Figure 1* Schematic plot of fatigue crack propagation rate ($da/dN$) against stress intensity range ($\Delta K$) depicting the three FCP rate regimes.

where $R$ is the load ratio (minimum to maximum), $\alpha$ is considered a constant, $q$ is the strain gradient which after Rice [6] may be taken equal to $1/(1 + n)$, $n$ being the cyclic strain hardening coefficient, $\Delta\varepsilon$ is the crack tip strain range, $X$ is the distance ahead of the crack tip, and $A$ and $B$ are fitting constants. Regardless of the process-damage accumulation or cycle by cycle growth that leads to the crack propagation, the element at $X_0$ fractures when $\varepsilon(X_0)$ attains a critical value.

With crack closure, there is a reduction in the crack tip driving force which in turn should lead to a reduction in the crack tip strain range from $\Delta\varepsilon$ to an effective value $\Delta\varepsilon_{\text{eff}}$ and an increase of the load ratio from $R$ to $R_c$ at the crack tip. By taking $\Delta\varepsilon = \Delta\varepsilon_{\text{eff}}$ and $R = R_c$, Equation 2 for crack closure condition can be written as

$$\varepsilon(X_0) = \frac{\alpha}{1 - R_c}\left(\frac{1}{R_{\text{p,eff}}}\right)^q \Delta\varepsilon_{\text{eff}} \tag{3}$$

where $R_{\text{p,eff}}$ is the CPZ size corresponding to $\Delta\varepsilon_{\text{eff}}$, which, in fact, represents the actual strain range required for crack propagation. From the fact that the FCP rate remains the same with respect to applied as well as effective driving force and as the FCP rate, in principle, should be proportional to the strain in the element at $X_0$, it is reasonable to consider that the incremental crack size ($X_0$) and hence $\varepsilon(X_0)$ are unaltered in the presence of crack closure.

It has been suggested [7] that the occurrence of crack closure leads to a strain intensification near the crack tip. This becomes apparent from the fact that higher applied strain ($\Delta\varepsilon$) is necessary to achieve $\Delta\varepsilon_{\text{eff}}$ at the crack tip in promoting crack growth. The magnitude of this strain intensification then can be obtained from Equations 2 and 3:

$$\Delta\varepsilon - \Delta\varepsilon_{\text{eff}} = \Delta\varepsilon\left[1 - \left(\frac{1 - R_c}{1 - R}\right)\left(\frac{R_{\text{p,eff}}}{R_{\text{p}}}\right)^q\right] \tag{4}$$

Relaxation of this strain intensification can occur either by enhancing the FCP rate or by promoting the plastic deformation to a greater extent. The latter is considered to be favoured since with crack closure the resistance to the FCP rate is actually increased in terms of the applied driving force. Close dependence of plastic deformation on shear strain suggests a correspondence, to a first approximation, between ($\Delta\varepsilon - \Delta\varepsilon_{\text{eff}}$) of Equation 4 and $2\Delta\gamma$, where $\Delta\gamma$ is the shear strain range. Note that this near-crack-tip shear strain directly results from the applied strain. Conceptually, $R_{\text{p}}$ (and by analogy, $R_{\text{p,eff}}$) is proportional to $\Delta K^2(\Delta K^2_{\text{eff}})$, $\Delta K\,[\Delta K_{\text{eff}}] = K_{\max}(1 - R)[K_{\max}(1 - R_c)]$, and as noted earlier, $q = 1/(1 + n)$. When these considerations are employed, Equation 4 becomes

$$\frac{\Delta\gamma}{\Delta\varepsilon} = \frac{1}{2}\left[1 - \left(\frac{\Delta K_{\text{eff}}}{\Delta K}\right)^{(3+n)/(1+n)}\right] \tag{5}$$

## 3. Results and discussion

Equation 5, therefore, gives an estimate of the fraction of the crack tip strain range that dissipates in shear

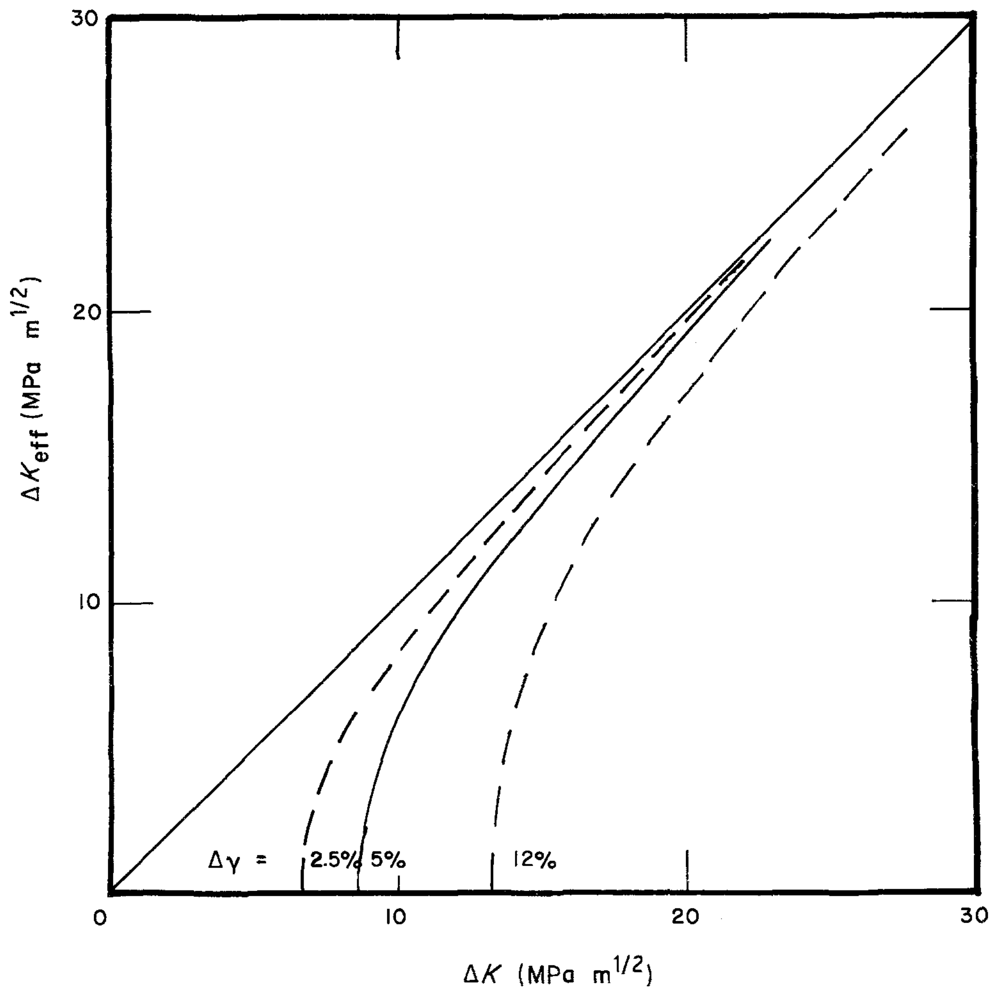


*Figure 2* Shear strain ($\Delta\gamma$) effect on the predicted variation of $\Delta K_{eff}$ with $\Delta K$ for a 7075-T6 aluminium alloy.

strain in the presence of crack closure. To obtain an estimate of this fraction, an experimental value of $\Delta K_{eff}/\Delta K = 0.48$ at the threshold level for a 2.25 Cr–1 Mo steel [8] is substituted in Equation 5 which yields $\Delta\gamma/\Delta\varepsilon = 0.43$ for an average value of $n = 0.15$ [9]. Thus, it is evident that a large fraction of the applied strain may be converted into shear strain, when crack closure is involved.

One could also consider Equation 5 as an estimate for driving force required for propagating a crack. Following this, it is apparent that if the FCP is assisted by a higher $\Delta\gamma$, then higher $\Delta K$ is necessary to achieve $\Delta K_{eff}$, as it is an increasing function of the FCP rate, required for promoting crack growth. Again, the close dependence of plastic deformation on shear strain makes it reasonable to suggest that an improvement in FCP resistance should follow an effort that promotes plastic deformation to a greater extent.

To facilitate the assessment of the plastic deformation, further simplification of Equation 5 is possible since

$$\Delta\varepsilon = C\Delta K^m, \quad (6)$$

where $C$ and $m$ are constants [10]. Combining Equations 5 and 6 gives

$$\Delta K_{eff} = \Delta K \left\{1 - 2\frac{\Delta\gamma}{C\Delta K^m}\right\}^{(1+n)/(3+n)} \quad (7)$$

a correlation between $\Delta K$ and $\Delta K_{eff}$ in terms of $\Delta\gamma$ by involving a few material constants. Equation 7 is depicted in Fig. 2 for various values of $\Delta\gamma$ for a 7075–T6 aluminium alloy with $C = 2.8 \times 10^{-4}$ and $m = 2.7$ [10]. It can be seen that at lower values of $\Delta K$, $\Delta K_{eff}$ is considerably lower than $\Delta K$. As $\Delta K$ is increased, this desparity is decreased. The effect of $\Delta\gamma$ is to further increase this desparity at lower $\Delta K$. At higher $\Delta K$, however, $\Delta K$ and $\Delta K_{eff}$ seem to converge regardless of the value of $\Delta\gamma$.

Clearly, the plastic deformation (i.e. $\Delta\gamma$) plays an important role in determining the FCP rate at lower $\Delta K$ levels approaching the threshold. The variables that can affect plastic deformation are the grain size, microstrucure, environment as well as $R$-ratio at a given temperature. The FCP rate dependence in region I and hence the $\Delta K_{th}$ values on the variables just noted are widely recognized; thus lending confidence to the above notion. The plastic deformation evidently resists the FCP at the lower $\Delta K$ levels and as a consequence, an effort to promote plastic deformation to a greater degree should improve the FCP kinetics in region I and, in turn, the $\Delta K_{th}$ values.

On the other hand, the effect of the degree of the plastic deformation on the FCP rates at higher values of $\Delta K$, where $\Delta K_{eff}$ approaches $\Delta K$, appears minimal. Such FCP rate insensitivities on the external variables such as microstructure, grain size, environment and $R$-ratio lead to the idea that the FCP rate at higher values of $\Delta K$ is governed by the continuum behaviour, as originally noted by Paris and Johnson [11], and, as a result, is sensitive only to $\Delta K$.

Furthermore, some of the FCP rate features of small cracks may also be rationalized using the idea presented here. For example, higher FCP rates of small cracks as opposed to large ones at an equivalent driving force can be attributed to the lower degree of plastic deformation ($\Delta\gamma$) required for their propagation. This is equivalent to saying that a lower amount of the applied crack tip strain range is dissipated in shear strain and as a consequence, $\Delta\varepsilon_{eff}$ is higher at the tip of a small crack than a large one. The crack tip opening displacement (CTOD) being a measure of crack tip deformation, the observation [12] that the CTOD is higher for a small crack than a large crack at an equivalent driving force is clearly in accord with the above argument.

Finally, one may invoke the FCP rate insensitivities on the plastic deformation ($\Delta\gamma$) as a possible criterion to explain the FCP rate merger of small and large cracks at a high $\Delta K$ value. Furthermore, it has been noted [12] that the FCP rates of small cracks are insensitive to the environments. Hence, in keeping with the present line of arguments, any shift of the FCP rate merger due to the effect of environment towards a lower or higher value of $\Delta K$ may accordingly be rationalized due to the need for a lower or higher degree of plastic deformation in promoting the growth of a large crack.

An important aspect of this work, as it is apparent from the above, is that it is concerned with how the localized plastic deformation controls the FCP kinetics. The variables such as grain size, microstructure, environment, $R$-ratio as well as crack size affect the FCP rate (particularly in region I) through their influence on the degree of the localized deformation. It is established here that the FCP rate is decreased, in general, as the plastic deformation required for crack propagation increases.

One could also consider the stability of the dislocation substructures resulting near the crack tip due to the localized plastic deformation ($\Delta\gamma$) in order to derive the additional validity of the above general trend. In view of the cyclic deformation studies [13–15], which have well established the unique dependence of the dislocation substructures on the shear strain level, the dislocation substructures typical of lower strain fatigue is expected to be present near the crack tip when its propagation is assisted by a lower $\Delta\gamma$. Then, energetic consideration suggests a lower degree of stability and, in turn, a higher FCP rate of the dislocation substructures, at an equivalent driving force, than that of higher strain fatigue.

## 4. Summary and conclusion

Recognizing the complexity associated with the FCP, an attempt is made here to evaluate the general dependence of the $\Delta K_{eff}$–$\Delta K$ disparity and, in turn, the FCP rate on the degree of plastic deformation ($\Delta\gamma$) by adopting a few simplified considerations of relevance to the crack closure phenomenon. The effect of plastic deformation is most significant at lower $\Delta K$ (region I approaching the threshold) levels than higher ones (region II). Accordingly, the FCP resistance in region I and, in turn, the $\Delta K_{th}$ value could be improved by promoting the plastic deformation at the crack tip to a greater degree. This approach could, therefore, form the basis to explain the grain size, microstructure, environment, $R$-ratio and crack size effects on the FCP rates. It should be emphasized that no attempt is made here to model the FCP rate. In view of this, the model presented can be considered qualitative. However, the FCP rate, crack tip strain, both tensile and shear (using a microtome [16, 17]) measurements and an effort to model the FCP rate which will provide experimental verification of this proposed idea is being undertaken in this laboratory and will be the subject of future publications.

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